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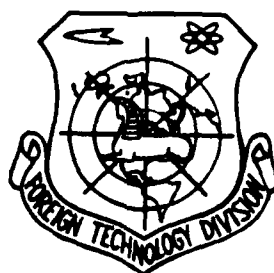


LAMINAR FLAME IN A TURBULENT FLOW

by

A.M. Klimov

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## PARTIALLY EDITED MACHINE TRANSLATION

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LAMINAR FLAME IN A TURBULENT FLOW

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# U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	<b><i>А а</i></b>	A, a	Р р	<b><i>Р р</i></b>	R, r
Б б	<b><i>Б б</i></b>	B, b	С с	<b><i>С с</i></b>	S, s
В в	<b><i>В в</i></b>	V, v	Т т	<b><i>Т т</i></b>	T, t
Г г	<b><i>Г г</i></b>	G, g	У у	<b><i>У у</i></b>	U, u
Д д	<b><i>Д д</i></b>	D, d	Ф ф	<b><i>Ф ф</i></b>	F, f
Е е	<b><i>Е е</i></b>	Ye, ye; E, e*	Х х	<b><i>Х х</i></b>	Kh, kh
Ж ж	<b><i>Ж ж</i></b>	Zh, zh	Ц ц	<b><i>Ц ц</i></b>	Ts, ts
З з	<b><i>З з</i></b>	Z, z	Ч ч	<b><i>Ч ч</i></b>	Ch, ch
И и	<b><i>И и</i></b>	I, i	Ш ш	<b><i>Ш ш</i></b>	Sh, sh
Й й	<b><i>Й й</i></b>	Y, y	Щ щ	<b><i>Щ щ</i></b>	Shch, shch
К к	<b><i>К к</i></b>	K, k	Ъ ъ	<b><i>Ъ ъ</i></b>	"
Л л	<b><i>Л л</i></b>	L, l	Ы ы	<b><i>Ы ы</i></b>	Y, y
М м	<b><i>М м</i></b>	M, m	Ь ь	<b><i>Ь ь</i></b>	'
Н н	<b><i>Н н</i></b>	N, n	Э э	<b><i>Э э</i></b>	E, e
О о	<b><i>О о</i></b>	O, o	Ю ю	<b><i>Ю ю</i></b>	Yu, yu
П п	<b><i>П п</i></b>	P, p	Я я	<b><i>Я я</i></b>	Ya, ya

\*ye initially, after vowels, and after ъ, ь; e elsewhere.  
When written as ѐ in Russian, transliterate as yě or ě.

## RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	sinh <sup>-1</sup>
cos	cos	ch	cosh	arc ch	cosh <sup>-1</sup>
tg	tan	th	tanh	arc th	tanh <sup>-1</sup>
ctg	cot	cth	coth	arc cth	coth <sup>-1</sup>
sec	sec	sch	sech	arc sch	sech <sup>-1</sup>
cosec	csc	csch	csch	arc csch	csch <sup>-1</sup>

Russian      English

rot      curl  
lg      log

## GRAPHICS DISCLAIMER

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## LAMINAR FLAME IN A TURBULENT FLOW.

A. M. Klimov.

(Novosibirsk).

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Are obtained equations, which describe laminar flame in heterogeneous hydrodynamic field if curvature of flame front can be disregarded (action of large-scale turbulence). The basic properties of this flame are clarified. The possible mechanism of extinguishing of flame is obtained. The mechanism of turbulent combustion is considered. *Flow fields, Russian translations. (Mylar)*

1. Heterogeneity of hydrodynamic field has fundamental importance for mechanism of turbulent combustion; we will consider turbulence as probability field of velocity gradients. Let us designate through  $l$  the size of the region, in which the velocity gradients for a certain period of time (lifetime of this region) weakly depend on the position of point within the region, although they can arbitrarily depend on time. This size is identical to the scale of turbulence. Let us introduce also the significant dimension  $\delta$  of "temperature" field, after defining it as distance in the direction of the gradient of the temperature (or concentration), on

which the temperature (concentration) substantially varies. In our case this will be the thickness of laminar flame in the homogeneous mixture. The character of phenomenon, obviously, strongly depends on the relationship between  $l$  and  $\delta$ . If  $l \gg \delta$  (large-scale turbulence), flame front weakly is bent ( $r \gg \delta$ , where  $r$  - radius of curvature of flame front), but the area of its surface continuously is changed: flame is stretched or, on the contrary, is contracted. Is investigated below precisely this case ( $l \gg \delta$ ).

Let us consider field of velocities of region  $l$  without combustion, which we will call external hydrodynamic field. The field of pure shear (Fig. 1a) is the simplest example, when the correlation between the velocity components is absent. Another example is "flow at critical point" (Fig. 1b), that appears, if, for example, in regions A and B rarefactions arose. The correlation between the velocity components here is exhibited in the pure form. The velocity field of arbitrary region  $l$  is a superposition of these two limiting cases. For example, in the plane case, taking into account the equation of continuity, velocity field takes the form

$$u^c = k(t)x - k_1(t)y, \quad v^c = -k(t)y - k_2(t)x$$

It is evident that to "flow at critical point"  $u=kx$ ,  $v=-ky$  are superimposed arbitrary shifts  $u=k_1y$ ,  $v=k_2x$ . The three-dimensional case in principle does not differ from the plane.

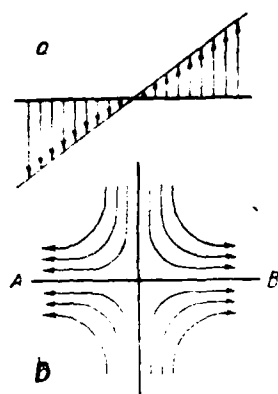


Fig. 1a, b.

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Assumptions, essential for obtaining equations, will be usual assumptions of theory of slow combustion: 1) pressure constancy (equation of motion is not examined), 2) in equation of energy is considered only thermal energy of gas.

Furthermore, we will consider only primary processes of transfer and consider that heat release rate is determined by concentration of one component of gas mixture and by temperature, and Lewis number is equal to 1. Then, as is known, the equation of diffusion it is possible to exclude.

In conventional designations in plane case we will have

$$\begin{aligned}
 \rho C_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) &= \\
 &= \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda \frac{\partial T}{\partial y} \right) + F(T) \\
 \frac{\partial \omega}{\partial t} + \frac{\partial (\omega u)}{\partial x} + \frac{\partial (\omega v)}{\partial y} &= 0 \quad \left( \begin{array}{l} \rho = \rho(T) \\ \lambda = \lambda(T) \end{array} \right) \quad (1.1)
 \end{aligned}$$

Furthermore, is assigned applied field  $u^0, v^0$ . Let the applied field - pure shear. Without the limitation of generality it is possible to consider  $u^0 = a(t)y, v^0 = 0$ ; the orientation of the initial distribution of temperature (being one-dimensional) relative to applied field is assigned by angle  $\psi$ , where  $\phi\phi$  - flame front (Fig. 2).

True hydrodynamic field will differ from external, since during combustion, as a result of thermal expansion of gases, motion, normal to flame front, appears. As a whole the phenomenon is such: flame front is turned by applied field in plane  $xy$  with a change in the area of its surface, simultaneously it moves relative to gas, creating additional velocity field due to the thermal expansion.

We seek solution of equations (1.1) in the form

$$T = T(t, \xi), \quad u = ay + V(t, \xi) \left[ 1 - \left( \operatorname{tg} \psi - \int_0^t a \dot{a} dt \right)^2 \right]^{-1/2},$$

$$r = V(t, \xi) \left[ \operatorname{tg} \psi - \int_0^t a \dot{a} dt \right] \left[ 1 + \left( \operatorname{tg} \psi - \int_0^t a \dot{a} dt \right)^2 \right]^{-1/2},$$

where

$$\xi = \left[ x - y \left( \operatorname{tg} \psi - \int_0^t a \dot{a} dt \right) \right] \left[ 1 + \left( \operatorname{tg} \psi - \int_0^t a \dot{a} dt \right)^2 \right]^{-1/2},$$

For determining  $T(t, \xi)$  and  $V(t, \xi)$  we will obtain equations

$$\begin{aligned} \rho C_p \left[ \frac{\partial T}{\partial t} + q(t) \xi \frac{\partial T}{\partial \xi} + V \frac{\partial T}{\partial \xi} \right] &= \frac{\partial}{\partial \xi} \left( \lambda \frac{\partial T}{\partial \xi} \right) + F(T) \\ \frac{\partial \rho}{\partial t} + q(t) \xi \frac{\partial \rho}{\partial \xi} + \frac{\partial (\rho V)}{\partial \xi} &= 0 \end{aligned} \quad (1.2)$$

Here

$$\varphi(t) = a \left[ \operatorname{tg} \psi - \int_0^t a dt \right] \left[ 1 + \left( \operatorname{tg} \psi - \int_0^t a dt \right)^2 \right]^{-1}$$

Coordinate  $\xi$  is always normal to the surface of flame.



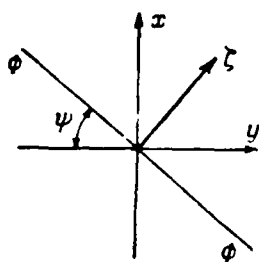


Fig. 2.

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Velocity  $V$  is connected with expansion of gas. If combustion products and combustible mixture occupy half-spaces, it is natural to consider  $V=0$  for the fresh mixture. Initial and boundary conditions for equations (1.2) in this case will be

$$T = T^*(\xi) \quad \text{when } t = 0, \quad V \rightarrow 0 \quad \text{with } \xi \rightarrow -\infty.$$

If at initial moment of time combustion products occupy the band of finite width with the symmetrical temperature distribution, the simplest initial and boundary conditions will be

$$T = T^0(\xi) \quad \text{when } t = 0, \quad \frac{\partial T}{\partial \xi} = V = 0 \quad \text{with } \xi = 0$$

Let now  $u^0 = -k(t)x$ ,  $v^0 = k(t)y$ . We seek the solution of equations (1.1) in the form

$$\begin{aligned} T &= T(t, \xi) \\ u &= -kx + V(t, \xi) \operatorname{ctg} \psi \exp\left(2 \int_0^t k dt\right) \left[1 + \operatorname{ctg}^2 \psi \exp\left(4 \int_0^t k dt\right)\right]^{-1/2}, \\ v &= ky + V(t, \xi) \left[1 + \operatorname{ctg}^2 \psi \exp\left(4 \int_0^t k dt\right)\right]^{-1/2}. \end{aligned}$$

where

$$\zeta = \left[ y + x \operatorname{ctg} \psi \exp \left( 2 \int_0^t k dt \right) \right] \left[ 1 + \operatorname{ctg}^2 \psi \exp \left( 4 \int_0^t k dt \right) \right]^{-1/2}$$

For determining  $T(t, \zeta)$  and  $V(t, \zeta)$  we will obtain (1.2), where

$$\varphi(t) = k \left[ 1 - \operatorname{ctg}^2 \psi \exp \left( 4 \int_0^t k dt \right) \right] \left[ 1 + \operatorname{ctg}^2 \psi \exp \left( 4 \int_0^t k dt \right) \right]^{-1}$$

In general case we will also obtain equations (1.2). The specific character of each case is included in function  $\varphi(t)$ .

It is not difficult to check that  $\varphi(t)$  is relative rate of change of surface area of flame, taken with opposite sign.

Equations (1.2) it is possible to also obtain, having assumed that flame remains plane and is assigned function  $\varphi(t)$ . Let  $\xi, \eta, \zeta$  - coordinate system, oriented so that the axis  $\zeta$  is always normal to the surface of flame, and let  $v_\xi, v_\eta, v_\zeta$  - corresponding velocities.

Since surface of flame is plane,

$$T = T(t, \zeta), \quad v_\zeta = v_\zeta(t, \zeta)$$

Consequently,

$$\begin{aligned} \rho C_p \left( \frac{\partial T}{\partial t} + v_\zeta \frac{\partial T}{\partial \zeta} \right) &= \frac{\partial}{\partial \zeta} \left( \lambda \frac{\partial T}{\partial \zeta} \right) + F(T) \\ \frac{\partial \rho}{\partial t} + \rho \left( \frac{\partial v_\xi}{\partial \xi} + \frac{\partial v_\eta}{\partial \eta} + \frac{\partial v_\zeta}{\partial \zeta} \right) + v_\zeta \frac{\partial \rho}{\partial \zeta} &= 0 \end{aligned} \quad (1.3)$$

Let us consider small element of surface  $d\xi d\eta$  in arbitrary plane  $\xi = \text{const}$ . Relative rate of change in its area

$$-\varphi(t) = \frac{1}{d\xi d\eta} \frac{d(d\xi d\eta)}{dt} = \frac{\partial v_\xi}{\partial \xi} + \frac{\partial v_\eta}{\partial \eta} \quad (1.4)$$

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After representing  $v_\xi$  in the form  $v_\xi(t, \xi) = \varphi(t) \xi + V(t, \xi)$  and taking into account (1.4), we will obtain from (1.3) equation (1.2).

Let us introduce dimensionless quantities

$$\begin{aligned} \tau &= tu_0 / \delta_0, & \beta &= \xi / \delta_0, & \omega &= V / u_0, & \theta &= (T - T_0) / (T_1 - T_0) \\ \rho' &= \rho / \rho_0, & \lambda' &= \lambda / \lambda_0, & f &= F / F_0, & \varepsilon &= \varphi \delta_0 / u_0 \quad (\delta_0 = \lambda_0 / \rho_0 u_0 C_p) \\ \rho_0 &= \rho(T_0), & \lambda_0 &= \lambda(T_0), & F_0 &= \rho_0 u_0 C_p (T_1 - T_0) / \delta_0 \end{aligned}$$

Here  $u_0$  - velocity of propagation of normal flame;  $\delta_0$  - distance, at which in normal flame  $\theta$  is changed approximately  $e$  times (exactly  $e$  times in solution of Michealson);  $T_0$ ,  $T_1$  - initial and final (adiabatic) temperatures of gas;  $F_0$  - average heat release rate in normal flame.

Parameter  $\varepsilon$  is ratio of characteristic time of smooth burning  $t_0 = \delta_0 / u_0$  to characteristic time of action  $t_1 = 1/\varphi$  on flame of external hydrodynamic field;  $t_1$  - time, for which with  $\varphi = \text{const}$  surface area of flame is changed  $e$  times, or, which is the same, time, within which distance between surfaces of constant temperature would be changed  $e$  times in the absence of chemical reaction and thermal conductivity.

In adopted designations system (1.2) takes form

$$\rho \left[ \frac{\partial \theta}{\partial \tau} + \varepsilon(\tau) \beta \frac{\partial \theta}{\partial z} + \omega \frac{\partial \theta}{\partial z} \right] = \frac{\partial}{\partial z} \left( \lambda \frac{\partial \theta}{\partial z} \right) + f(\theta) \quad \frac{\partial \rho}{\partial \tau} + \varepsilon(\tau) \beta \frac{\partial \rho}{\partial z} + \frac{\partial (\rho u_z)}{\partial z} = 0 \quad (1.5)$$

Here and throughout primes with  $\rho$  and  $\lambda$  are omitted.

Now, in comparison with equations of one-dimensional flame, terms in (1.5) are of order  $\varepsilon(\tau)$ . Let us do numerical estimations. Let, for example, the velocity of the turbulent pulsations  $v' \sim 10$  m/s with the scale of turbulence  $l \sim 1$  cm; then  $t_l \sim 10^{-3}$  s. For the majority of the fuel-air mixtures, close to stoichiometric,  $t_0 \sim 10^{-3}$  s and rapidly increases to  $10^{-2}$  s during impoverishment or enrichment of mixture approximately 1.5 times. Respectively, we will obtain  $\varepsilon \sim 1-10$ , depending on the composition of mixture.

Parameter  $\varepsilon$  is in close agreement with parameter  $\gamma = v' \delta_0 / u_0 l$ , proposed [1] for evaluation of relative role of "surface" and "volumetric" combustion. The difference between them is that in  $\gamma$  value  $v'/l$  is the averaged velocity gradient, and value  $\phi$  in  $\varepsilon$  - instantaneous and local. Therefore  $\varepsilon$  can be both positive and negative value. It is evident from (1.4) that the gradients of different velocity components can make different contribution to a change in the surface area of flame - flame can be stretched or contracted in all directions, but it can be contracted in one direction and be stretched in another.

Let us write equations (1.5) in coordinate system, connected with flame front, i.e., in system of coordinates, whose origin is connected with certain temperature  $\theta_0$ . The coordinate  $\beta_0$  of point with the temperature  $\theta_0$  is determined by the equation

$$\frac{d\beta_0}{d\tau} = [\varepsilon(\tau)\beta_0 - \omega] - [m(\tau) + \omega] = \varepsilon(\tau) - m(\tau)$$

Consequently,

$$\beta_0(\tau) = \exp\left[\int_0^\tau \varepsilon(\tau) d\tau\right] \left[\beta_{00} - \int_0^\tau m(\tau) \exp\left(-\int_0^\tau \varepsilon(\tau) d\tau\right) d\tau\right] \quad (3\omega = 3_0(0))$$

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In variables  $\alpha = \beta - \beta_0$ ,  $\tau$  system (1.5) will take form

$$\begin{aligned} \rho \left[ \frac{\partial \theta}{\partial \tau} + (m + \varepsilon \alpha + \omega) \frac{\partial \theta}{\partial \alpha} \right] &= \frac{\partial}{\partial \alpha} \left( \lambda \frac{\partial \theta}{\partial \alpha} \right) + f(\theta) \\ \frac{\partial \rho}{\partial \tau} + (m + \varepsilon \alpha) \frac{\partial \rho}{\partial \alpha} + \frac{\partial (\rho \omega)}{\partial \alpha} &= 0 \end{aligned} \quad (1.6)$$

Function  $m(\tau)$  is determined during solution of specific problem. The initial value  $m(\tau)$  must be assigned.

Equations (1.6) can be also obtained from equations (1.5), by considering that latter are obtained from (1.3), where  $\xi$ ,  $\eta$ ,  $\zeta$  - coordinate system, connected with flame front. It is necessary only to represent  $\omega$  in the form

$$\omega(\beta, \tau) = m(\tau) + \omega_1(\beta, \tau), \quad m(\tau) = \omega(-\alpha, \tau)$$

2. Before constructing solution of problem, let us give qualitative description of picture of phenomenon in question.

In uniform hydrodynamic field ( $\epsilon=0$ ) solution of system (1.6) under some assumptions relative to function of heat release  $f(\theta)$  and corresponding initial and boundary conditions approaches time-independent solution and which describes temperature distribution in normal flame. In this case  $m(\tau) \rightarrow 1$ . The stationary form of normal flame is a result of the equilibrium of opposite "forces": the smoothing out action of thermal conductivity is balanced by the opposite action of chemical reaction.

With  $\epsilon \neq 0$  appears new factor - convective heat transfer  $\epsilon \alpha \partial \theta / \partial x$ , which either smooths out ( $\epsilon > 0$ ), or it increases ( $\epsilon < 0$ ) slope of temperature distribution.

If  $\epsilon \rightarrow \text{const} < 0$ , solution of system (1.6) must approach certain, steeper than normal flame, stationary temperature distribution.

Different proves to be the situation with  $\epsilon > 0$ . In the model of gaseous medium accepted the thermal disturbances are propagated with infinite velocity. In practice this velocity is also very great (speed of sound). Therefore the thermal effect of hot region instantly penetrates to large distance into the cold region. At the same time at the sufficiently large distance from the origin of

coordinates to the side of cold gas (i.e. with the sufficiently large negative  $\alpha$ ) the convection heat current acquires the same direction as the heat flow, caused by thermal conductivity. During the slow combustion such heat transfer into the cold region is compensated in no way; conditions at the large distance from the flame front always vary and entire process becomes unsteady.

For quantity  $q$  of heat separating per unit of surface of flame per unit time (proportional to quantity of burning substance), taking into account that  $\partial\theta/\partial\alpha \rightarrow 0$  with  $\alpha \rightarrow \pm\infty$ , relationship

$$q = \int_{-\infty}^{+\infty} j dx = \int_{-\infty}^{+\infty} \rho \frac{\partial\theta}{\partial x} - \int_{-\infty}^{+\infty} \rho (m + \epsilon x + \omega) \frac{\partial\theta}{\partial x} dx$$

In normal flame  $q = \rho(m + \omega) = \text{const.}$  In remaining cases ( $\epsilon \neq 0$  or  $\epsilon = 0$ , but phenomenon is unsteady)  $q$  is not connected unambiguously with  $\rho(m + \omega)$  and, generally speaking, it is variable.

In time intervals, when applied field vanishes, flame is one-dimensional, but unsteady.

Structure of this flame has much in common with flame structure with  $\epsilon \neq 0$ ; therefore let us consider it in more detail.

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In one-dimensional unsteady flame  $m + \omega$ , being velocity of motion of temperature  $\theta$ , relative to gas ( $m$  - relative to fresh mixture,  $m + \omega$

- relative to particles of gas with other temperatures), can take any values, in particular negative. If  $m+\omega < 0$ , gas flows into this plane  $\alpha$ -const not from the side of fresh mixture, but from the side of combustion products. The flow of molecules (as heat flow) is composed of the flows of convective and diffusion. In the normal flame convective and diffusion flows of fuel are combined. In the unsteady flame they can be subtracted. Selecting conditionally the zone of reaction, it is possible to encounter the situation, when gas flows into this zone from the side of combustion products, and fuel is supplied towards the convection current by diffusion. Let, for example, initial function for equation (1.5) be

$$\theta(\beta, 0) = 0 \quad \text{when } \beta < 0, \quad \theta(\beta, 0) = 1 \quad \text{with } \beta > 0 \quad (2.1)$$

before will be established the normal flame, in which any temperature moves towards the fresh mixture, for a certain period of time the points with the temperature, close to  $\theta=1$ , will move towards the combustion products. Consequently, at least in the part of the zone of reaction, the convection current of fuel (in the system of coordinates  $\alpha$ ) will be directed against the diffusion. The establishment of normal flame in this case occurs by means of the leveling action of thermal conductivity and diffusion.

If function  $\theta(\beta, 0)$ , on the contrary, is very flat in comparison with case of normal flame, main factor of establishment of normal flame will be local increase in temperature due to chemical reaction. In this case the velocity of motion relative to gas of points with



temperatures, close to  $\theta=1$ , at the first moments of time will be considerably more than this velocity in the normal flame, having the same direction.

It follows from equations (1.6) that in heterogeneous hydrodynamic field the concept of velocity of propagation of flame front, strictly speaking, does not make sense. With any  $\epsilon \neq 0$  the velocity of the motion of gas relative to the fixed temperature  $\theta$ , varies in the range  $(-\infty, +\infty)$ , depending on the temperature of the particles of the gas. Certainly, with small  $|\epsilon|$  this concept can make approximate sense. Hydrodynamic field in the system of coordinates of flame appears on "flow at critical point" mentioned above with the plane of symmetry, parallel to flame front. With small  $|\epsilon|$  this plane is located far from the region with the high gradients of temperature (region of basic part of the change in the temperature from zero to one); the field of the mass flow rate in the latter is close to uniform and flame differs little from the one-dimensional. With the sufficiently large  $|\epsilon|$  the plane of symmetry is moved into the region with the high temperature gradients and the concept of the velocity of propagation of flame front does not have even approximate sense.

3. Integration of first equation (1.6) with respect to  $\tau$  gives

$$\theta(\alpha, \tau) = \theta(\alpha, 0) - \int_0^\tau (m + \epsilon\alpha + \omega) \frac{\partial \theta}{\partial \alpha} d\tau + \int_0^\tau \frac{\partial}{\partial \alpha} \left( \lambda \frac{\partial \theta}{\partial \alpha} \right) d\tau + \int_0^\tau f d\tau$$

If flame was normal at initial moment, for small  $\tau$  with  $\epsilon = \text{const}$

we will obtain

$$\theta(\alpha, \tau) = \theta(\alpha, 0) - \epsilon \alpha \tau \frac{\partial \theta(\alpha, 0)}{\partial \alpha} + O(\tau^2)$$

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In normal flame  $\partial \theta / \partial \alpha \geq 0$ ; consequently, temperature distribution begins to be smoothed out with values  $\epsilon > 0$  and to become steeper with  $\epsilon < 0$ .

We investigate behavior of flame with large  $|\epsilon|$ . In this case the temperature distribution everywhere, with exception of the narrow layer of thickness  $h \sim 1/\sqrt{|\epsilon|}$  in the vicinity of point  $\alpha=0$ , is described by the equation

$$\frac{1}{\epsilon} \frac{\partial \theta}{\partial \tau} + \alpha \frac{\partial \theta}{\partial \alpha} = 0 \quad (3.1)$$

which is obtained from (1.6) by the rejection of small terms.

This equation possesses system of characteristics  $\mu = \alpha \exp(\epsilon \tau)$ , along which  $\theta = \text{const.}$

Let  $\epsilon < 0$ . Then it follows from (3.1) that for the time  $\tau \sim 1/|\epsilon|$  almost all the change in the temperature will be concentrated in layer  $h$ . With  $|\epsilon| \rightarrow \infty$  we have  $h \rightarrow 0$ , and the temperature profile approaches disruptive <sup>1</sup>.

FOOTNOTE <sup>1</sup>. If we take into account thermal conductivity, for the

temperature distribution we will obtain

$$\theta = \frac{1}{2} + \frac{1}{2} \operatorname{erf} \left( \sqrt{\frac{1}{2}} |z| \alpha \right) \quad (-\infty \leq z \leq +\infty)$$

passing with  $\epsilon \rightarrow -\infty$  in (2.1), if we in (2.1) by  $\beta$  imply  $\alpha$ . ENDFOOTNOTE.

With  $\epsilon > 0$  (3.1) gives "deliquescence" of flame with the same characteristic time  $\tau \sim 1/\epsilon$ ; distance between points with any two temperatures grows proportionally to  $\exp(\epsilon \tau)$ , respectively everywhere decreases gradient of temperature.

In order to obtain solutions with final  $|\epsilon|$ , we will use function  $f(\theta)$ , which corresponds to infinite zone of reaction:

$$f(\theta) \geq 0, \quad f(1) = 0, \quad f(\theta) = 0 \quad \text{with} \quad \theta < 1, \quad \int_0^1 f(\theta) d\theta = \int_0^1 f_0 d\theta \quad (3.2)$$

where  $f_0$  - true function of heat release. Let

$$f(\theta) \geq 0 \quad \text{when} \quad 0 < \theta_0 \leq \theta \leq 1, \quad f(\theta) = 0 \quad \text{with} \quad \theta < \theta_0, \quad f(\theta_0) = f(1) = 0$$

Then, if  $\theta_0 \rightarrow 1$  and  $f(\theta)$  approaches function (3.2), rightly, as in the case of normal flame [2], the relationship

$$\lim_{\theta_0 \rightarrow 1} \left[ \frac{\partial \theta}{\partial z} \right]_{z=0} = \left[ \frac{2}{\lambda(1)} \int_0^1 f d\theta \right]^{1/2} \quad (3.3)$$

Let us prove it. From equations (1.6) we have

$$-\frac{\rho^2}{d\rho/d\theta} \frac{\partial \omega}{\partial z} = \frac{\partial}{\partial z} \left( \lambda \frac{\partial \theta}{\partial z} \right) + f(\theta) \quad (3.4)$$

Hence

$$\begin{aligned}
 - \left[ \frac{\rho^2}{d\theta} \right]_{\theta_1}^{\theta_2} \int_{\theta_1}^{\theta_2} \frac{\partial \omega}{\partial \theta} d\theta &= \int_{\theta_1}^{\theta_2} \frac{d\lambda}{d\theta} \left( \frac{\partial \theta}{\partial x} \right)^2 d\theta + \int_{\theta_1}^{\theta_2} \lambda \frac{\partial \theta}{\partial x} d \left( \frac{\partial \theta}{\partial x} \right) + \int_{\theta_1}^{\theta_2} f(\theta) d\theta = \\
 &= \left( \frac{d\lambda}{d\theta} \right)_{\theta_1} \left[ \theta_2 \left( \frac{\partial \theta}{\partial x} \right)_{\alpha=0}^2 - \theta_1 \left( \frac{\partial \theta}{\partial x} \right)_{\alpha=0}^2 \right] - \lambda(\theta_2) \frac{1}{2} \left( \frac{\partial \theta}{\partial x} \right)_{\alpha=0}^2 + \int_{\theta_1}^{\theta_2} f d\theta
 \end{aligned}$$

where  $\theta_0 \leq \theta_1$ ,  $\theta_1$ ,  $\theta_2$ ,  $\theta_3 \leq 1$ . In obtaining (3.4) was used the integration by parts and was used condition  $\partial \theta / \partial \alpha = 0$  with  $\theta = 1$ .

Function  $\omega$  (gas velocity, connected with thermal expansion) must have limited derivative  $\partial \omega / \partial \alpha$ .

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Completing in (3.4) passage to the limit with  $\theta_0 \rightarrow 1$ , we will obtain (3.3). Let us note that in this case

$$q = \text{const} = \lambda(1) \left[ \frac{\partial \theta}{\partial x} \right]_{x=0}$$

Thus, using function (3.2), we will have  $\theta = 1$  with  $\alpha \geq 0$ ,  $\partial \theta / \partial \alpha = \text{const}$  with  $\alpha = -0$ . In the field  $[-\infty, 0]$  the function  $\theta$  is described by equations (1.6) with  $f=0$ .

We will seek steady-state solution of these equations with  $\epsilon = -c = \text{const} < 0$ . For obtaining the qualitative picture we will be bounded to approach  $\rho = \lambda = 1$ . Taking into account that  $\partial \theta / \partial \alpha = 1$  with  $\alpha = 0$ , let us find

$$\theta = \int_{-\infty}^{\alpha} e^{-c\alpha'} e^{m\alpha'} d\alpha' \quad (-\infty \leq \alpha \leq 0)$$

Value  $m$  is determined from condition

$$\int_{-\infty}^0 e^{-c\alpha^2/2 + m\alpha} d\alpha = 1$$

In particular

$$m = 1 \quad \text{при } c = 0, \quad m = 0 \quad \text{при } c = \pi/2$$

$$m \sim -\sqrt{2c \ln Vc} \quad \text{при } c \rightarrow \infty$$

Key: (1). with.

Solutions for different  $c$  are given in Fig. 3.

With  $c > \pi/2$  we have  $m < 0$ , i.e., gas flows into zone of reaction (point  $\alpha=0$ ) from side of combustion products. Curve  $\theta(\alpha)$  at point  $\alpha=0$  has positive curvature with  $m > 0$  and negative with  $m < 0$ . On the section of convexity of this curve the gas velocity  $m=c\alpha$  is negative and diffusion flow of fuel is directed against the convective.

Sequence of curves  $\theta(\alpha)$  with change of  $c$  from  $\infty$  to 0 is qualitatively identical to sequence of curves during establishment of normal flame from disruptive profile of temperature (2.1).

With  $\epsilon > 0$  steady-state solution of equations (1.6), which satisfies conditions  $\theta(-\infty)=0$ ,  $\theta(+\infty)=1$ , does not exist. The physical causes for this were explained in p. 2.

However, with  $\epsilon \ll 1$  it is possible to obtain approximate

steady-state solution, after introducing fictitious heat of sink with  $\theta=0$ . If the power of the heat of sink is small in comparison with the power of heat source, it is possible to consider that the temperature distribution will evolve slowly and for a certain period of time will differ little from the solution with the sink.

Using (3.2), with  $\rho=\lambda=1$  in stationary case we will obtain

$$\frac{d\theta}{dx} = e^{m\alpha + \epsilon x^2/2} \quad (3.5)$$

Heat sink  $q_0$  will be minimum, if it is located at point  $\alpha_0 = -m/\epsilon$  ( $q_0 = \exp(-m^2/2\epsilon)$ ). Boundary conditions for (3.5)

$$\theta = 1 \quad \text{when} \quad x = 0, \quad \theta = 0 \quad \text{with} \quad x = -m/\epsilon$$

Consequently,

$$\theta = \int_{-m/\epsilon}^x e^{m\alpha + \epsilon \alpha^2/2} d\alpha \quad (-m/\epsilon < x \leq 0)$$

Condition for determining  $m$  will be

$$\int_{-m/\epsilon}^0 e^{m\alpha + \epsilon \alpha^2/2} d\alpha = 1 \quad (3.6)$$

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During solution of (3.6) relative to  $m$  appears ambiguity, easily, however, removable by condition  $m=1$  with  $\epsilon=0$ .

We give results of calculations of  $m$  and  $\alpha_0$  for some values of  $\epsilon$

$\epsilon =$	0	0.097	0.18	0.26	0.40	0.516	0.57	0.58
$m =$	1.0	1.10	1.20	1.26	1.26	1.22	1.07	1.0
$-\alpha_0 =$	$\infty$	11.3	6.67	4.85	3.16	2.36	1.88	1.72

Curves  $\theta(\alpha)$  are given in Fig. 3. Close to reality will be the solutions, apparently, only to  $\epsilon \approx 0.2$ , when the value of sink is still small ( $q_0 = \exp(-2)$  with  $\epsilon = 0.18$ ). With  $\epsilon > 0.3$  the effect of sink is so considerable that  $m$  begins to decrease. The solution does not exist with  $\epsilon > 0.58$  (with  $\epsilon = 0.58$  we have  $q_0 \approx 1/e$ ). Thus, with  $\epsilon \sim 1$  phenomenon becomes essentially unsteady.

4. Use of function (3.2) does not make it possible to find change of  $q$ . For solving this question it is necessary to use zone of reaction of finite width. Nevertheless, the explained general properties of phenomenon make it possible to draw qualitative conclusions without the exact solution of problem. Actually

$$q = \int_{-\infty}^{+\infty} f d\alpha = \int_0^1 f \frac{\partial \alpha}{\partial \theta} d\theta$$

With  $\epsilon < 0$  in region of basic part of change of temperature (and, consequently, in basic part of zone of reaction)  $\partial \alpha / \partial \theta$  is reduced in comparison with normal flame, with  $\epsilon > 0$  - it increases. In other words, is narrowed down or is widened the zone of reaction; respectively,  $q$  is reduced or increased.

It is obvious that effect of  $\epsilon$  on  $q$  will be greater, the wider the zone of reaction in normal flame.

If we consider quantity  $Q$  of heat, which is isolated per unit of primary surface of flame, then it is possible to show that with time  $Q$  begins to increase with  $\epsilon < 0$  and to be reduced with  $\epsilon > 0$ , if at initial moment the flame was normal. With  $\epsilon < 0$  in the stationary case  $Q \sim q_{\text{exp}}(c\tau)$ . In the turbulent flow the stretching of flame ( $\epsilon < 0$ ) statistically predominates above contraction ( $\epsilon > 0$ ). This directly follows from the form of the function  $\phi(t)$ , but it can be considered also as experimental fact. Result - increase in rate of combustion in the presence of turbulence.

In flame with chain mechanism of chemical reaction the change of thickness of flame will influence diffusion of active centers and, apparently,  $q$  will be changed more slowly than with thermal mechanism of chemical reaction.

Is possible also change in reaction mechanism: decrease of thickness of flame will shift mechanism in direction of chain, increase - in direction of thermal.

5. If  $f(\theta) = 0$  with  $\theta \leq \theta_0 > 0$ ,  $f(\theta) \geq 0$  with  $\theta_0 \leq \theta \leq 1$ , then with sufficiently large  $c$  there is no different from zero steady-state solution of equations (1.5), which satisfies boundary conditions

$$\theta \rightarrow 0 \text{ when } \beta \rightarrow \pm \infty, \quad \frac{\partial \theta}{\partial \beta} = \omega = 0 \text{ with } \beta = 0$$



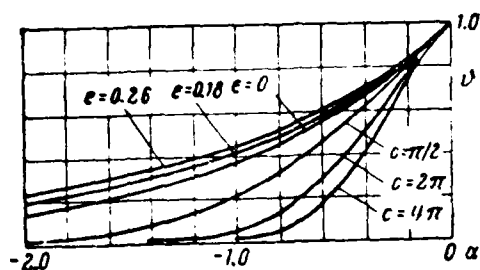


Fig. 3.

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I.e. with sufficiently strong and prolonged elongation finite volumes of combustion products, surrounded by fresh mixture, can vanish. The process of such extinguishing of flame is very simple. Let at initial moment of time the combustion products occupy in the plane  $\xi$  the band of finite width  $d \gg \delta_0$ . With sufficiently large  $c$  the contraction of band under the action of external hydrodynamic field cannot be compensated by the process of combustion and the bandwidth begins to be reduced. If in this case is reached  $d \sim \delta_0$ , the effect of cold regions is closed and occurs the temperature decrease, accompanied by end of burning.

6. Investigation conducted above makes it possible to make some conclusions about mechanism of turbulent combustion (in the case of large-scale turbulence).

Important role plays parameter  $\gamma = v' \delta_0 / u_0 l$ . Order of magnitude  $\gamma$ , obviously, coincides with the probable value of the absolute value of parameter  $\epsilon = \phi \delta_0 / u_0$ .

With  $\gamma \ll 1$  combustion occurs in laminar fronts. Perceptible deviations of flame from the normal and the phenomena, connected with extinguishing of flame (resorption of finite volumes of combustion products, discontinuities of flame, the throwing of combustion products into the combustible mixture), are unlikely.

With  $\gamma \sim 1$  structure of laminar flames, as a rule, differs from normal: concept of velocity of propagation of flame front becomes meaningless, temperature profile and quantity (specific) of burning substance are variable. The probability of extinguishing of flame increases; but, taking into account that for the possibility of extinguishing of flame  $|\epsilon|$  must be sufficiently large ( $c > \pi/2$  in the idealized diagram of p. 3) and is necessary some time, the greater it is, the less  $|\epsilon|$ , this probability should be recognized still small. Thus, with  $\gamma \sim 1$  combustion occurs predominantly in the "frontal" mode; the contribution of the "volumetric" combustion, connected with extinguishing of flame, is small.

With  $\gamma \gg 1$  structure of laminar flames strongly differs from normal, probability of extinguishing of flame becomes considerable. Combustion occurs both in laminar fronts, which constantly burst and extinguish, leaving "warm" regions, and with the help of volume reactions, which go in "warm" regions and constantly are accompanied by ignition with formation of laminar flame.

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